The basic problem of the calculus of variations: new range of validity of the Du Bois - Reymond equation and applications to regularity

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Abstract

The talk is centred on the basic problem of the calculus of variations

$$\min \int_a^b L(t, x(t), x'(t))\, dt,\ x \in W^{1,1}([a,b]),$$
$$x(a) = A, x(b) = B, x(t) \in \Sigma \subset R^n. \tag{P}$$

Several applied problems involve Lagrangians that are far from satisfying the classical Tonelli's existence assumptions: they happen to be not regular in x, x', non convex in x' and may even not be superlinear.

Under more regularity assumptions, Lipschitz regularity of the minimizers or at least the nonoccurrence of the Lavrentiev phenomenon (i.e, the existence of a Lipschitz minimizing sequence) has been widely investigated.

In a joint work with P. Bettiol (Univ. Brest-France) we established the validity of the Du Bois - Reymond equation for a wide class of Lagrangians, containing the autonomous one that are *just* Borel; for scalar problems ($n = 1$) this particular case was prefigured in [4].

Several slow growth conditions introduced in the last decades by F. Clarke and A. Cellina appear to be a violation of the Du Bois - Reymond equation for high values of the velocity, and yield some new Lipschitz regularity results on the minimizers, whenever they exist, or just on the minimizing sequences, and provide some new sufficient conditions to exclude the Lavrentiev phenomenon in the nonautonomous case.

As a byproduct, we obtain a new Lipschitz regularity on the value function

$$V(t,x) := \inf \left\{ \int_t^T L(s, x(s), x'(x))\, ds + g(x(T)) : \; x(t) = x \right\}.$$

References

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